

Group A

Attempt Any Two: (2 x 10 = 20)

1. Define the following three measures of dispersion – range, standard deviation and inter quartile range –by clearly state their properties. Write down a situation where range is preferred to Standard deviation. Score obtained by 10 Students in a test are given below. Compute range, and standard deviation.

42	55	35	60	55	55	65	40	45	35
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2. There are three traffic lights on your way home. As you arrive at each light assume that it is either red (R) or green (G) and that it is green with probability 0.7 . Construct the sample space by listing all possible eight simple events. Assign probability to each simple event. Are the events equally likely? What is the probability that you stop no more than one time.
3. A large company wants to measure the effectiveness of radio advertising media (X) on the sale promotion (Y) of its products. A sample of 22 cities with approximately equal population is selected for study. The sales of the product in thousand Rs and the level of radio advertising expenditure in thousand Rs are recorded for each of the 22 cities (n) and sum, sum of square, and sum of cross product of X and Y are summarized below.

$$\sum Y = 26953, \sum X = 950, \sum Y^2 = 35528893, \sum X^2 = 49250, \& \sum XY = 1263940$$

- (a) Fit a simple linear regression model of Y on X using the least square method. Interpret the estimated slope coefficient.
- (b) Compute R^2 and interpret it.

Group B

Answer any eight questions: (8 x 5 = 40)

4. Describe the scopes and limitations of statistics in empirical research.
5. Write down the properties and importance of density function of a continuous random variable. Suppose a continuous random variable X has the density function.

$$f(x) = \begin{cases} k(1-x)^2 & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find (a) value of the constant k, and (b) $E(X)$.

6. Suppose that X and Y have joint density function

$$f(x) = \begin{cases} (x+y), & \text{if } 0 < x, \quad y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find

(a) Marginal density function of X and Y, and

(b) Covariance between X and Y.

7. In a Poisson distribution with parameter λ derive the mean and variance of the distribution.
8. The length of life of automatic washer (X) is approximately normally distributed with mean and standard deviation equal to 3.1 and 1.2 years, respectively. Compute the probabilities (a) $P(X>1)$, (b) $P(X>2.5)$ and (c) $P(1<X<2)$.
9. If $X_1, X_2, \dots, X_n$ are n independent random variables each is distributed as normal with mean μ and variance σ^2 , then derive the distribution of $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$

10. If a continuous random variable X has exponential distribution with density function

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

For $h>0$, prove that $P(X>t+h | X>t) = P(X>h)$, and hence prove that

$$P(X>t+h) = P(X>t) \times P(X>h).$$

11. If $X_1, X_2, \dots, X_n$ are n independent Bernoulli random variables with common mean p, derive the maximum likelihood estimator of p. Prove or disprove the estimator is unbiased for p?
12. A car manufacturer claims that its car use, on average, no more than 5.5 gallons of petrol for each 100 miles. A consumer groups tests 40 of the cars and finds an average consumption of 5.65 gallons per 100 miles and a standard deviation of 1.52 gallons. Do these results cast doubt on the claim made by the manufacturer? Answer the question by setting appropriate null and alternative hypotheses and testing the null hypothesis at 5% level of significance.
13. The average length of time required to complete a certain aptitude test is claimed to be no more than 80 minutes. A sample of 25 students yielded an average of 86.5 minutes and a standard deviation of 15.4 minutes. Do these results cast doubt on the claim? Assuming that test score is normally distributed answer the query by setting appropriate null and alternative hypotheses and testing the null hypothesis at 5 % level of significance.